Convergent Adaptive Discretization Methods for Computing Correlated Equilibria of Polynomial Games (Extended Abstract)

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In this paper, we develop an adaptive discretization method for computing correlated equilibria in n-player games with polynomial utilities. To date most research on computing equilibria has considered only finite games. Our focus is on algorithmically computing exact and approximate correlated equilibria in games with infinite strategy spaces. In previous work [1] we characterized correlated equilibria of continuous games and suggested a number of algorithms for computation. We also presented a natural heuristic for producing a sequence of finitely supported ϵ-correlated equilibria (described below) which in practice seems to drive ϵ to zero quickly.

Our first result in this paper shows that this procedure does not always drive ϵ to zero, i.e., lead to an exact correlated equilibrium in the limit, even in finite or polynomial games. The bulk of this paper develops an alternative, though still intuitive and simple, algorithm for computing a sequence of finitely supported ϵ-correlated equilibria in games with infinite strategy spaces. We show that this algorithm converges (i.e. $\epsilon \to 0$) for games with compact metric strategy spaces and continuous utilities. For games with polynomial utilities, we show how to use sum-of-squares and semidefinite programming techniques to efficiently implement these algorithms.

Under the simplest choice of parameters the convergent algorithm proceeds as follows. The k^{th} iteration begins with a finite subset $\tilde{C}_i^k$ of player i's pure strategies for each i; the initial choice of strategies $\tilde{C}_i^0$ is arbitrary. We then compute an ϵ^k-correlated equilibrium π^k supported on the product $\prod \tilde{C}_i^k$. This distribution π^k is chosen to minimize ϵ^k subject to the condition (and this is where we differ from the nonconvergent heuristic) that π^k is an exact correlated equilibrium of the finite game induced when deviations are restricted to $\tilde{C}_i^k$.

We must then use π^k to enlarge $\tilde{C}_i^k$ to a new set $\tilde{C}_i^{k+1}$. To do so, we select some player i for whom the ϵ^k-correlated equilibrium constraints are tight. This player can improve his payoff by ϵ^k by switching from his recommendations under the distribution π^k to some other strategies in his pure strategy space C_i. By the assumption that π^k is a correlated

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equilibrium of the game when deviations are restricted to $\tilde{C}_i^k$, at least one of these other strategies must lie outside of $\tilde{C}_i^k$. We add these strategies to $\tilde{C}_i^k$ to produce $\tilde{C}_i^{k+1}$. For all other players $j \neq i$, we set $\tilde{C}_j^{k+1} = \tilde{C}_j^k$. Finally we let $k = k + 1$ and iterate.

If we remove the condition that π^k be a correlated equilibrium when deviations are restricted to $\tilde{C}_i^k$, this procedure need not converge as can be seen by considering the following finite symmetric game with identical utilities.

	a	b	c
a	0	1	0
b	1	5	7
c	0	7	0

If we let $\tilde{C}_i^0 = \{a\}$ for both players then π^0 must assign all probability to (a, a) and we get $\epsilon^0 = 1$. Then $\tilde{C}_i^1 = \{a, b\}$ for both players, but it can be shown that the optimal π^1 is in fact $\pi^1 = \pi^0$. Therefore the procedure gets "stuck" so $\tilde{C}_i^k$, π^k and $\epsilon^k = 1$ remain constant for all $k \geq 1$. Adding the condition that π^k be a correlated equilibrium when deviations are restricted to $\tilde{C}_i^k$ ensures that the algorithm can explore the strategy spaces enough to not get stuck in this way.

We leave open the problem of determining how to direct this algorithm toward a correlated equilibrium with particular desirable properties, such as one which maximizes the sum of utilities.

References

[1] N. D. Stein, P. A. Parrilo, and A. Ozdaglar. Characterization and computation of correlated equilibria in infinite games. In *Proceedings of the 46th IEEE Conference on Decision and Control*, pages 759 – 764, 2007.