Airline Network Design and Hub Location Problems[*]

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Abstract

Due to the popularity of hub-and-spoke networks in the airline and telecommunication industries, there has been a growing interest on hub location problems and related routing policies.

In this paper, we introduce flow-based models for designing capacitated networks and routing policies. No a priori hub-and-spoke structure is assumed. The resulting networks may suggest the presence of "hubs", if cost efficient.

The network design problem is concerned with the operation of a single airline with a fixed share of the market. We present three basic integer linear programming models, each corresponding to a different service policy. Due to the difficulty of solving (even small) instances of these problems to optimality, we propose heuristic schemes based on mathematical programming.

The procedure is applied and analyzed on several test problems consisting of up to 39 U.S. cities. We provide comments and partial recommendations on the use of hubs in the resulting network structures.

Keywords: capacitated network design problem, hub-and-spoke system

1 Introduction

Since the 1978 Airline Deregulation Act, perhaps the most significant innovation in the airline industry has been the adoption of hub-and-spoke systems. Flights from different origins to a same destination , or from a same origin to different destinations are consolidated via intermediate nodes called hubs. Hubs exploit economies of scale by allowing a smaller number of higher capacitated arcs to serve a large number of origin-destination pairs. The concept has also been applied to telecommunication networks.

Past studies on hub-and-spoke networks in the airline industries have appeared both in the area of airline economics (e.g., Bailey, Graham and Kaplan, 1985, Brown, 1991, Drezner, 1994, Hansen and Kanafani, 1990, Morrison and Winston, 1986, Reynolds and Feighan, 1992) and in the area of operations research/transportation science (e.g., Aykin, 1994, 1995a, 1995b, Campbell, 1994a, 1994b, 1994c, Ernst and Krishnamoorthy, 1996, O'Kelly, 1987, O'Kelly, Skorin-Kapov and Skorin-Kapov, 1995, Skorin-Kapov and Skorin-Kapov, 1994, Skorin-Kapov, Skorin-Kapov and O'Kelly, 1995). Our paper falls in this second category, which we now briefly review.

Campbell (1994a) provides a nice survey of network hub location problems. A classical and most frequently addressed problem is the single allocation p-hub median problem with non-hub routes prohibited. Given that there must be p hubs in the network, find an optimal set of locations such that each non-hub city is connected to a single hub, while the total transportation cost to serve a specified set of flows is minimized. Flows between nodes are generally routed via one or (at most) two hubs. O'Kelly (1987) presents a quadratic integer program for the problem as well as two heuristics. The procedures are tested on a data set consisting of air passenger traffic in the United States in 1970 as evaluated by the Civil Aeronautics Board (CAB). Following this seminal paper, many papers have appeared on this problem. For example, Skorin-Kapov and Skorin-Kapov (1994) use tabu search in order to get some of the best solutions for the CAB data, Campbell (1994c) proposes two new heuristics based on greedy interchange, O'Kelly, Skorin-Kapov and Skorin-Kapov (1995) develop lower bounds for hub location problems, and Skorin-Kapov, Skorin-Kapov and O'Kelly (1995) and Ernst and Krishnamoorthy (1996) propose exact solution methods based on tight linear programming relaxations.

Multiple allocation version of the p-hub median problem allows a non-hub city to be connected to more than one hub. Campbell (1994b) presents integer programming formulations for a variety of single and multiple allocation hub location problems, and introduces

hub center and hub covering problems. O'Kelly and Miller (1994) present various classes of hub location problems corresponding to different decisions on the allocation type, non-hub routes and hub level network topology.

In an attempt to introduce a more comprehensive framework with networking policies, Aykin (1994, 1995a, 1995b) develops several integer programming models for single allocation and multiple allocation hub location problems. Two networking policies are considered: (i) Nonstrict hubbing, in which channeling flows through hubs is not required but chosen if efficient and (ii) strict hubbing, in which all flows to/from a node are channeled through the same hub. In both cases, at most two-hub-stop services are allowed. Aykin's models also include fixed costs for establishing hubs. The proposed solution procedures include enumeration algorithms and greedy-interchange heuristics, along with Lagrangian relaxation-based lower bounds.

All models discussed above are based on the following rationale: Economies of scale due to hubbing are explicitly and a priori modeled by having inter-hub transportation cost discounted by a constant factor $0 \leq \alpha \leq 1$, or, in more elaborate models, by three constant factors $\alpha_1, \alpha, \alpha_2 \in [0, 1]$, for spoke-to-hub, inter-hub, and hub-to-spoke links, respectively. In addition, Aykin's more elaborate models also indirectly consider the impact on aircraft loading, and thus on revenue. Flights between hub cities are assumed to have a constant 80% load factor (percentage of seats filled by revenue paying passengers), and flights between hub and spoke cities a 60% average load factor. Consequently, in all these models, there is no need to keep track of the number of passengers on each flight, and to make decisions on aircraft types, and the number of aircraft of each type in order to meet the demand (i.e. to provide enough capacity on each arcs). Finally all the models generally assume that the total flow of a given origin-destination pair will be served via a single path only (obtained as an output of the models).

In this paper we propose a radically different approach for the design of airline networks. First we do not assume a priori a hub-and-spoke structure, and thus our models do not consist of locating a given fixed number of hubs (if consolidation of flights through a given city is cost efficient, the models are intended to exhibit this behavior). Second, our models track the number of passengers on a given flight, and involve the choice of different aircraft types of different capacity and of the number of aircraft of each type to meet the demand (the impact of economies of scale on cost and load factor is an output of the model and will vary across pairs of cities). Third, our models allow many different paths between a given origin-destination pair (due to capacity constraints or opportunities for consolidation

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and economies of scale, a fraction of the demand may go on a direct flight, another fraction might have a one-stop flight, etc.). These three points considerably change the nature of the problem, and a direct quantitative comparison with the previous models would be meaningless (cost structures, objective functions, and decision variables are different). However we will test our algorithms on the CAB data and see if both approaches suggest similar cities as hubs (definition of a hub in this paper will be discussed later).

In Section 2, we introduce the generic design problem of interest, and present three different service policies. We then describe various mixed integer programming formulations. In Section 3, we present and test heuristic algorithms. In Section 4 we analyze the resulting networks. Finally, in Section 5 we provide concluding remarks.

2 Description and Formulation of the Problems

We consider the operation of a single airline with a fixed share of the market. The generic network design problem is stated as follows:

> Given a fixed origin-destination flow demand matrix, the capacity and mileage cost of different types of aircraft, design a network and a routing policy which satisfy the demand and minimize the total transportation cost.

Policy Classification: The following three service policies are considered.

> **One-Stop**: Under this policy, the airline provides two possible services for each route it serves: i) non-stop flight and ii) one-stop connecting flight.
>
> **Two-Stop**: Similar to the one-stop case, except that the airline now provides an additional service: iii) two-stop connecting flight.
>
> **All-Stop**: Under this policy, no restriction is imposed on the number of connecting stops.
>
> The two-stop case is the most common type of policy practiced in the U.S. airline industry today. The all-stop case has various important applications in telecommunication, air cargo delivery, and other logistical systems. It also serves as a benchmark (lower bound on the optimal value) for the other two policies.

Input for the models:

Network: Let N be the set of all cities. Let $d_{ij} = d_{ji}$ be the air distance between city i and city j.

Demand: Let f_{ij} be the flow, i.e. the number of passengers who desire to fly from city i to city j per day. We will generally assume that $f_{ij} = f_{ji}$.

Supply: Let K be the set of different types of aircraft to chose from. For each aircraft of type $k \in K$, let c_k be the cost per mile, and b_k be the capacity.

2.1 Formulation of the One-Stop Model

Let x_{ilj} be the fraction of the flow f_{ij} from i to j served by a one-stop connecting flight through city l, and let y_{ij}^k be the number of aircraft of type k used on the arc from city i to city j. The model can be formulated as follows:

(One-Stop)
$$\min \sum_{i \neq j} \sum_{k \in K} d_{ij} c_k y_{ij}^k$$

$$\text{s.t.} \quad f_{ij} + \sum_{t \neq i,j} \{ f_{it} x_{ijt} + f_{tj} x_{tij} - f_{ij} x_{itj} \} \leq \sum_{k \in K} b_k y_{ij}^k \quad \text{for all } i \neq j \quad (1)$$

$$\sum_{t \neq i,j} x_{itj} \leq 1 \quad \text{for all } i \neq j \quad (2)$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j \quad (3)$$

$$y_{ij}^k \geq 0 \quad \text{and integer for all } i \neq j,\ k \in K. \quad (4)$$

The objective function represents the total transportation costs. Constraints (1) are capacity constraints saying that the total fractional flow on arc (i,j) cannot exceed the aircraft total capacity assigned to that arc. Constraints (2) ensure that $1 - \sum_{t \neq i,j} x_{itj}$, the fraction of the flow f_{ij} from i to j served by a direct flight, is non-negative.

The following three points are not taken into account in this simplified model: (i) No fixed cost for purchasing/leasing aircraft, (ii) No limit on the total number of available aircraft, and (iii) No explicit periodic airline operations. The first two points can easily be incorporated into our formulation. An explicit consideration of the third one would require a significant modification of the model. Instead, we implicitly address this issue by assuming a symmetric OD demand matrix, an assumption commonly made in practice by most airlines while assessing their network strategies.

In conclusion we believe that the above model captures the essence of our purpose, i.e. to see if economy of scale would lead to a network structure with "hubs", and if not, to

see what alternative structure(s) it leads too. The model also integrates network design, aircraft choices, and routing policies.

2.2 Formulation of the Two-Stop Model

The formulation of this model is a simple extension of the previous one. In addition to the previous variables, let x_{iltj} be the fraction of the flow f_{ij} from i to j served by a two-stop connecting flight through cities l and t. We then have:

$$\textbf{(Two-Stop)} \qquad \min \sum_{i \neq j} \sum_{k \in K} d_{ij} c_k y_{ij}^k$$

$$s.t. \quad f_{ij} + \sum_{t \neq i,j} \{f_{it} x_{ijt} + f_{tj} x_{tij} - f_{ij} x_{itj}\} + \sum_{l,t \neq i,j} \{f_{lj} x_{ltij}$$

$$+ f_{it} x_{ijlt} + f_{lt} x_{lijt} - f_{ij} x_{iltj}\} \leq \sum_{k \in K} b_k y_{ij}^k \quad \text{for all } i \neq j$$

$$\sum_{t \neq i,j} x_{itj} + \sum_{l,t \neq i,j} x_{iltj} \leq 1 \quad \text{for all } i \neq j$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j$$

$$x_{iltj} \geq 0 \quad \text{for all } i \neq j \neq l \neq t$$

$$y_{ij}^k \geq 0 \quad \text{and integer for all } i \neq j,\ k \in K.$$

2.3 Formulation of the All-Stop Model

By letting S_{ij} be the set of all paths from city i to city j, and by defining new variables x_P for each path $P \in S_{ij}$, we could extend the previous formulation. However, since a path may involve up to n cities, this would not be practical (exponential number of real variables).

A more reasonable formulation is to consider a multicommodity network flow model. Let D be the set of all origin-destination pairs, and for each $d \in D$, let $O(d), D(d)$ and f_d be the origin node, the destination node, and the demand, respectively.

Let z_{ij}^d be be the fraction of the flow f_d routed through arc (i,j). The formulation becomes:

$$\textbf{(All-Stop)} \qquad \min \sum_{i \neq j} \sum_{k \in K} d_{ij} c_k y_{ij}^k$$

$$s.t. \quad \sum_{j \neq i} z_{ij}^d - \sum_{j \neq i} z_{ji}^d = \begin{cases} f_d & \text{if i=O(d)} \\ -f_d & \text{if i=D(d)} \quad \text{for all } i \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$\sum_{d \in D} z_{ij}^d \leq \sum_{k \in K} b_k y_{ij}^k \quad \text{for all } i \neq j \tag{6}$$

$$z_{ij}^d \geq 0 \quad \text{for all } i \neq j, \ d \in D \tag{7}$$

$$y_{ij}^k \geq 0 \quad \text{and integer for all } i \neq j, \ k \in K. \tag{8}$$

Constraints (5) correspond to the usual flow conservation, and constraints (6) model the arc capacity.

Remarks:

1. This formulation is very close to the network loading problem introduced by Mirchandani (1989), see also Magnanti, Mirchandani and Vachani, 1993, 1995.

2. This arc-based formulation is obviously more compact, but the model does not give explicit (path) routing policies for the demand (i.e $x's$). Instead the $x's$ need to be "reconstructed" from the $z's$ (e.g. see Ahuja, Magnanti and Orlin, 1993).

2.4 Transformation of the All-Stop Model into a One-Stop Model

In Song, 1995, a transformation of the all-stop model into a variation of the one-stop model is presented. It is as follows:

1. Let k^* be the most efficient type of aircraft, i.e. minimizing c_k/b_k.

2. Choose m large so that $mb_{k^*} \geq \sum_{ij} f_{ij}$.

3. Add mb_{k^*} to each f_{ij}, getting new OD demand $\bar{f}_{ij} = mb_{k^*} + f_{ij}$.

4. Solve the one-stop model under the new OD demand and with the additional constraints:
$$y_{ij}^{k^*} \geq m \quad \text{for all } i \neq j.$$

5. Let $\{\hat{y}_{ij}^{k^*}, (\hat{y}_{ij}^k)_{k \neq k^*}\}$ be an optimal solution to this new one-stop model, then $\{\hat{y}_{ij}^{k^*} - m, (\hat{y}_{ij}^k)_{k \neq k^*}\}$ is an optimal solution to the original all-stop model.

The formal proof is lengthy and technical, and out of context here, see Song, 1995 for details. Rather, let us give some intuition behind the transformation. First, it is relatively easy to show that the five steps above hold if applied to the all-stop model (i.e., replacing in the five steps 'one-stop' by 'all-stop'). Second, for this transformed all-stop model, the added flows

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and triangle inequalities allow "swapping arguments" that transform any optimal solution into one that never uses more than one connecting stops, i.e., a solution to the transformed one-stop model.

With this transformation, the all-stop model becomes no harder than the transformed one-stop model. On the other hand all models discussed here are strongly NP-hard (Song, 1995). Several exact solution procedures have been proposed to solve these models (Song, 1995), including a special procedure based on Benders decomposition and valid inequalities. Even though the most sophisticated of these exact methods significantly improve over classical Branch and Bound techniques, the size of the solvable problems remains very modest (less than 10 cities at best!). This is consistent with results on the network loading problems reported elsewhere (Magnanti, Mirchandani and Vachani, 1993, 1995, Mirchandani, 1989). These problems are *extremely* difficult to solve to optimality, even for very small instances.

3 Heuristic Procedures

From now on, due to space limit considerations, we are going to restrict ourselves to problems with either one type of aircraft (one-fleet option) or two types of aircraft (two-fleet option). We first consider in detail the solution procedure for the all-stop model with the one-fleet option, and then consider its modifications for the two-fleet option, and then for the one-stop and two-stop models. In a last subsection we present computational testing for all the heuristics.

3.1 All-Stop Model With One-Fleet Option

From Section 2.4, we know that we can consider this equivalent transformed one-stop formulation:

$$\textbf{(MIP)} \qquad \min \quad \sum_{i \neq j} d_{ij} c y_{ij}$$

$$s.t. \quad \bar{f}_{ij} + \sum_{t \neq i,j} \{\bar{f}_{it} x_{ijt} + \bar{f}_{tj} x_{tij} - \bar{f}_{ij} x_{itj}\} \leq b y_{ij} \quad \text{for all } i \neq j$$

$$\sum_{t \neq i,j} x_{itj} \leq 1 \quad \text{for all } i \neq j$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j$$

$$y_{ij} \geq m \text{ and integer for all } i \neq j.$$

The heuristic is a mathematical programming-based procedure using valid inequalities and local improvements. It consists of three major steps:

(1) finding an initial network structure ($y's$ feasible solution),

(2) improving it by local rules to a near optimal structure,

(3) obtaining routing flows (i.e., the $x's$ solution).

The second step is the most involved and consists of many different types of local improvements. Let us describe each step in more detail.

3.1.1 Step 1: Initial Feasible Solution

The LP relaxation of (MIP) provides a poor quality lower bound, and the LP solution, when rounded up to the nearest feasible integer solution, a poor starting solution. This is consistent with other capacitated network design problems (see e.g. Magnanti, Mirchandani and Vachani, 1995). We then add the following valid inequalities to (MIP), hereafter called *one-demand cuts*:

$$\sum_{j \neq i} y_{ij} \geq \lceil \sum_{j \neq i} \bar{f}_{ij}/b \rceil \quad \text{for all } i \in N,$$

Let $\{y_{ij}^*\}$ be the corresponding solution to the new LP relaxation. The starting initial feasible integer solution for (MIP) is then defined as $\bar{y}_{ij} = \lceil y_{ij}^* \rceil$.

3.1.2 Step 2a: Accommodating Paths

Step 2a attempts to decrease the number of aircraft on arcs, by shifting some of their flow to currently over-capacitated arcs. It uses the following idea.

Let r_{ij} be the "residual capacity" and t_{ij} be the fraction flow on arc (i, j) corresponding to the initial solution, i.e., $r_{ij} = \bar{y}_{ij} - y_{ij}^*$, and $t_{ij} = 1 - r_{ij}$. For a given OD pair ij, the set of paths $\{(i, t_k, j)\}_k$ is called *accomodating* if $\sum_k \min\{r_{it_k}, r_{t_kj}\} \geq t_{ij}$. For such a set, we can redirect the fraction flow t_{ij} from the arc (i, j) to the set of paths $\{(i, t_k, j)\}_k$ and thus remove an aircraft from arc (i, j).

Step 2a searches for sets of accomodating paths in the following order: It scans once every arc (i, j) such that $\bar{y}_{ij} > 0$ in non-increasing order of d_{ij}. For each arc, the procedure adds each path $\{(i, t_k, j)\}_k$ to a set in non-increasing order of $\min\{r_{it_k}, r_{t_kj}\}$ until either the set becomes accomodating, or there are no more such path to add.

3.1.3 Step 2b: Arc Interchanges

Step 2b attempts to move an aircraft from a long arc to a short one. It uses the following idea.

Suppose that for a set of three nodes i, k, j we have $r_{ij} > 0$, $r_{ik} > 0$. If $d_{ij} > d_{kj}$ and $t_{ij} + r_{ik} \leq 1$, then, by redirecting the fraction flow t_{ij} from the arc (i, j) to the path (i, k, j), we achieve an improvement of $d_{ij} - d_{jk}$ by setting $\bar{y}_{ij} := \bar{y}_{ij} - 1$, $\bar{y}_{kj} := \bar{y}_{kj} + 1$.

Step 2b scans every arc (i, j) such that $r_{ij} > 0$ in non-increasing order of d_{ij}. It then considers each node k in non-decreasing order of d_{jk} until the above conditions hold or all nodes k have been considered.

3.1.4 Step 2c: Accommodating Paths, Bis

Same as step 2a, except that four-node paths are now considered.

3.1.5 Step 2d: Removing Arcs

This last local improvement stage uses a systematic approach for further reduction in the number of aircraft. Linear programming problems are solved in order to re-direct flows and maximize residual capacity on each arc. If the resulting residual on any arc (i, j) is $r_{ij} \geq 1$, we set $\bar{y}_{ij} := \bar{y}_{ij} - 1$. We then "remove" the arc from further consideration. We consider two succesive implementations of these ideas.

First, we solve the following linear programming model in order to maximize the total sum of weighted residuals for all arcs. We set a weight $w_{ij} = 0$ if $\bar{y}_{ij} = m$, d_{ij} otherwise.

$$\textbf{(LP1)} \qquad \max \quad \sum_{i \neq j} w_{ij} r_{ij}$$

$$s.t. \quad b\bar{y}_{ij} - \left\{ \bar{f}_{ij} + \sum_{t \neq i, j} (\bar{f}_{it} x_{ijt} + \bar{f}_{tj} x_{tij} - \bar{f}_{ij} x_{itj}) \right\} \geq b r_{ij} \quad \text{for all } i \neq j$$

$$\sum_{t \neq i, j} x_{itj} \leq 1 \quad \text{for all } i \neq j$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j$$

$$r_{ij} \geq 0 \quad \text{for all } i \neq j.$$

Given the current solution $\bar{y}_{ij}$, the x-feasible region is defined by the same constraints as in (MIP). Hence the problem feasibility is maintained. Upon obtaining the solution, we "remove" all arcs with $r_{ij} \geq 1$. If any arc is being removed and the solution is improved,

we reset the weights of the objective function and repeat this step until no improvement is achieved.

In the second implementation, the same linear programming model is solved, except that instead of maximizing the sum of residuals, we concentrate on one arc (i, j) at a time such that $r_{ij} > 0$ and $\bar{y}_{ij} > m$. The objective function becomes max r_{ij}.

3.1.6   Step 3: Final Network Structure and Routing Flows

A final y-solution, $\{\bar{y}_{ij}\}$, has now been constructed for (MIP). The final step constructs the y-solution and x-solution for the all-stop model.

By setting $\hat{y}_{ij} = \bar{y}_{ij} - m$ and $f_{ij} = \bar{f}_{ij} - mb$, we find a feasible z-solution, i.e such that:

$$\textbf{(Routing)} \tag{9}$$

$$\sum_{j \neq i} z_{ij}^d - \sum_{j \neq i} z_{ji}^d = \begin{cases} f_d & \text{if i=O(d)} \\ -f_d & \text{if i=D(d)} \quad \text{for all } i \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$\sum_{d \in D} z_{ij}^d \leq b\hat{y}_{ij} \quad \text{for all } i \neq j \tag{11}$$

$$z_{ij}^d \geq 0 \quad \text{for all } i \neq j \text{ and all } d \in D. \tag{12}$$

Finally from the arc flow z-solution we reconstruct the path flow x-solution using techniques described in Ahuja, Magnanti and Orlin, 1993.

3.2   All-Stop Model With Two-Fleet Option

Let the two types of aircraft be such that $b_1 > b_2$, $c_1 > c_2$, and $c_1/b_1 < c_2/b_2$ (type 1 is a larger aircraft with a larger operating cost per mile, but more efficient than type 2). (MIP) becomes:

$$\textbf{(MIP2)} \qquad \min \quad \sum_{i \neq j} d_{ij}(c_1 y_{ij}^1 + c_2 y_{ij}^2)$$

$$s.t. \quad \bar{f}_{ij} + \sum_{t \in N} \{\bar{f}_{it}x_{ijt} + \bar{f}_{tj}x_{tij} - \bar{f}_{ij}x_{itj}\} \leq b_1 y_{ij}^1 + b_2 y_{ij}^2 \quad \text{for all } i \neq j$$

$$\sum_{t \neq i,j} x_{itj} \leq 1 \quad \text{for all } i \neq j$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j$$

$$y_{ij}^1 \geq m \text{ and integer for all } i \neq j$$
$$y_{ij}^2 \geq 0 \text{ and integer for all } i \neq j.$$

Any optimal solution of the linear programming relaxation will give $y_{ij}^2 = 0$ for all (i,j). Based on this property, we design the heuristic so that a higher priority is given to type 1 aircraft.

Our approach consists of two stages: First we design the network as if only type 1 aircraft were available, applying the heuristic described in 3.1. Next we consider type 2 aircraft and try to find for each arc the best combination of the two aircraft types. For this last stage, an enumeration method is found to be efficient because of the relative small number of aircraft given by the solution.

3.3   One-Stop Model

The one-stop formulation with one type of aircraft becomes:

$$\min \quad \sum_{i \neq j} d_{ij} c y_{ij} \tag{13}$$

$$s.t. \quad f_{ij} + \sum_{t \neq i,j} \{f_{it} x_{ijt} + f_{tj} x_{tij} - f_{ij} x_{itj}\} \leq b y_{ij} \quad \text{for all } i \neq j \tag{14}$$

$$\sum_{t \neq i,j} x_{itj} \leq 1 \quad \text{for all } i \neq j \tag{15}$$

$$x_{itj} \geq 0 \quad \text{for all } i \neq t \neq j \tag{16}$$

$$y_{ij} \geq 0 \text{ and integer for all } i \neq j. \tag{17}$$

The heuristic is a modification of the all-stop algorithm presented in 3.1 so that one-stop requirements are maintained. In the following, we briefly describe these modifications.

3.3.1   Initial Feasible Solution

Nothing needs to be changed here. However our computational testing indicates that one is better off by not adding the one-demand cuts before the LP relaxation. We have a restricted feasible space for local improvements due to the one-stop restrictions, and adding these cuts may lead to a local optima from which it is harder to escape.

3.3.2 Local Improvements

We call an arc (i, j) *assigned* if it carries flow other than from f_{ij}. A fraction flow from an assigned arc may not be redistributed (interchanged) to other arcs, since it could violate the one-stop restriction. Thus, we only redirect a fraction flow through an accommodating path if it is from an arc that (1) has not been assigned; or (2) such that its portion of direct flow is larger than the fraction flow. In the latter case we can redistribute the portion of the direct flow instead of its fraction flow.

In the all-stop case, we design the heuristic so that it searches four-node paths for further reduction. We cannot do that here.

In the last stage of removing arcs, the procedure used in the all-stop case still applies with minor modifications.

Finally, the modifications to the previous algorithm for the two-fleet case parallel what we have seen for the all-stop case, and are thus not repeated here.

3.4 Two-Stop Model

Our heuristic is constructed similarily to the all-stop heuristic. Let us present the main differences.

3.4.1 Initial Feasible Solution

The initial feasible solution is the heuristic solution to the one-stop model.

3.4.2 Local Improvements

Local improvements go through the same steps as in 3.1 with the following modifications. With the two-stop option, channeling demand flow through a four-node path is now possible.

Also , in a one-stop solution, there exists a large number of three-node paths through which fraction flows are redistributed. If there exists a three-node accommodating path with respect to an assigned arc with $t_{ij} > 0$, then it would be feasible to redistribute the fraction flow t_{ij} through the accommodating path under the two-stop policy. The portion of the flow being carried on arc (i, j) is now going through a four-node path, and the other portion would simply go through a three-node path.

Finally, for the "removing arcs" step (see Section 3.1.5), the corresponding linear programming problems can be very large (the number of real variables is $O(n^4)$), and for some

of the larger instances, we use column reduction by looking only at a subset of possible "arc reductions".

Again, the modifications of the previous algorithm for the two-fleet case parallel what we have seen for the all-stop case.

3.5 Numerical Testing of the Heuristics

3.5.1 Construction of the test problems

Our approaches are first tested on the CAB data (see for example O'Kelly, 1987). This data set originates from the Civil Aeronautics Board and consists of 25 cities with their flow volumes and co-ordinates.

We also provide a new data set which we now describe. Among the largest (in population) 100 U.S. cities, we have selected a total of 39 cities. These cities have been chosen in such a way that all major geographical areas of U.S. are covered. The distances between cities correspond to air distance (see Fitzpatrick and Modlin, 1986,). Intercity passenger travel demand are estimated based on the following simple gravity model:

$$f_{ij} = \alpha \ (p_i p_j)^{0.5}, \tag{18}$$

where p_i is the population of city i and α a given constant. The actual population figures are obtained from the 1994 census. The data are summarized in Table 1.

Table 1 is about here

Based on the same set of 39 cities, we have tested the heuristics on six different demand levels by changing the parameter α in the gravity model. In Table 2 we give the lowest and highest possible flow for each of the six levels. In all cases, lowest demand flow is between Columbia and Des Moines, and the highest between New York City and Los Angeles.

Table 2 is about here

With respect to the fleet characteristics, we have chosen aircraft with $b_1 = 180$ and $c_1 = 1$ for the one-fleet option. For the two-fleet option, we have added aircraft with $b_2 = 100$ and $c_2 = 0.65$ (note that $c_1/b_1 \leq c_2/b_2$).

3.5.2 Computational Results

We present a summary of our test results in Table 3 for all seven problems (CAB and the six flow levels for the 39-city problem).

The policy column refers to the three connecting flight policies that we have considered in this paper. The lower bound column corresponds to the values of the linear programming relaxations. The next two columns give the cost values of our solutions under a one-fleet and two-fleet option, respectively. The next columns provide the gap between the heuristic solutions and the linear programming lower bound (heuristic/lower bound - 1). The CPU1 and CPU2 columns represent the CPU time (in seconds on a SPARCstation 10) under the one-fleet and two-fleet option, respectively.

Connecting Policy Impacts: For each of the seven problems the gap between the heuristic solution and the LP lower bound decreases as we go from the one-stop, two-stop to all-stop policies. It is more difficult to coordinate flow under the one-stop and, to a lesser extent, two-stop restrictions. Also for each of the seven problems, the two-stop policy is the most time consuming, the one-stop policy being the fastest for the problems CAB, 1, and 2, and the all-stop policy being the fastest for the other problems.

Flow Level Impacts: The heuristics work well for high demand volumes, i.e., problems 3, 4, 5, and 6, (they all are within 7% from optimality in the worst case and the average gap is 3.3%). However they do not seem to perform uniformly well under the lower volume scenarios. The all-stop policy remains very good, with a worst gap of 12.8% (for problem 1). On the other hand, the one-stop policy can lead to a significant degradation, e.g problem 1 under the one-fleet option has a gap of about 103%. However for problems with low demand the LP lower bound might not be tight at all. For example, the gap of the initial feasible solution for this problem was more than 500%.

To stress the importance of the level of the demand on the quality of our heuristics, note that if we further increase the demand level to obtain a lowest flow of 45 (instead of 20 in Problem 6), we obtain gaps of less than 0.5% under all options. As for the CAB data, if we consider annual flow instead of daily ones, the gaps would be reduced to less than 0.05% under all policies! Finally, as the demand density increases, the stop limitations become less restrictive and the solution differences between the three policies become negligible.

Fleet Option Impacts: The two fleet option yields better solutions than the single fleet, as expected. However, the gap improvement is minimal, with an average cost reduction of 2.3%, (of 1% if we discard problem 1 with a cost reduction of 28%). We have tested various

relative ratios of c/b and obtained the same results. A possible reason, and the one we tend to endorse in this paper, is that the consolidation of flow (by swapping and/or redirecting flow) obtained by our procedures leave little room for further improvements by a multi-fleet option. The addition of a second type of fleet does better for the one-stop and two-stop policies, especially at the lower end of the demand density, but the advantage diminishes as the demand rate increases.

Table 3 is about here

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4   Analysis of Network Structure

4.1   Solution Analysis

Due to space limitation, results are presented in detail for the two-stop model under the one-fleet option, both for the CAB data and the 39-city data under flow level 1 (1-39 data). Results from the other test sets are only summarized.

4.2   Criteria for the Analysis

In the following analysis, a city will be considered a potential hub candidate if the network structure and resulting flow pattern indicate that it plays a connecting role in a "significant" manner. In order to make this term more precise, we have considered different measures:

Plane: The number of aircraft flying out of a city.

ExtraPlane: The difference between Plane and the minimum number that would be needed for satisfying the demand of that city only.

OriPassgr: The total number of passengers originating from a city.

ExtraPassgr: The number of passengers using a city as a connecting stop.

PropDirPassgr: The proportion of total passengers originating from a city and traveling directly (no connecting stop) to their final destinations.

Clearly ExtraPlane and ExtraPassgr are the most natural candidates related to the notion of a "connecting" city. The other measures are related to the size of the cities and are included for testing purposes.

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It is important to stress that the notion of a hub as implicitly "defined" above differs significantly from the classical notion as used in the literature on the p-hub median problem and uncapacitated hub location problem. In this paper, hubs, or connecting cities, may arise as a natural consequence of the network structure and flows. In the classical literature, hubs correspond to cities with explicit and exclusive a priori economies of scale. Thus any cities with extra passengers, or extra planes, would necessarily be hubs.

Finally when comparing results, one should not forget that our models allow service policies with non-stop service.

4.2.1 CAB, Two-Stop Model, One-Fleet Option

From Table 4, three cities (Baltimore, Cincinnati, and Memphis) dominate the others with respect to both ExtraPlane and ExtraPassengr. These cities are not among the cities with large originating demand (see OriPassgr), but are relatively centrally located. On the other hand "big" cities like Los Angeles and New York (high OriPassgr) have some of the smallest values for ExtraPlane and ExtraPassenger. Finally PropDirPassgr doesn't seem to be a clear measure for defining hubs either(see Baltimore).

Table 4 is about here

Most remarks remain valid for the 5 other combination of connecting policy and fleet option tested on the CAB data. Table 5 summarizes the list of corresponding hub candidates.

Table 5 is about here

Although the set of potential hubs slightly varies, cities like Cincinnatti and Memphis are consistently at the top of the list.

It is interesting to compare our results on the CAB data with those obtained on the p-hub median problems. Table 6 gives some results obtained by Skorin-Karov, Skorin-Karov and O'Kelly (1995). Clearly, contrary to our main results, the p-hub median problems seem to favor big cities such as New York and Los Angeles. Cities like New York and Los Angeles were also obtained on a different set of data by Aykin (1995b), with a model closer to ours (two-stops, direct flights allowed), but still with a classical notion of hubs.

Table 6 is about here

4.2.2　1-39, Two-Stop Model, One-Fleet Option

The main remarks made on the networks obtained on the CAB data remain valid for the 39-city problem. As indicated in Table 7, three cities (Columbus, Kansas City, and St.Louis) come out as strong hub candidates. The criterion OriPassgr is again clearly imperfect as a hub measure (see e.g. cities 19-Los Angeles and 26-New York). Again, the geographic position of a city plays an overwhelming role.

Table 7 is about here

Let us summarize the results for the other tests on the 39-city problems:

Hub locations, if any, remain in the central regions.

As the demand level becomes higher, the network structures converge to the same topology, irrespective of the connecting policy.

As the demand density increase, the proportion of passengers traveling without connecting flights increases as well.

In comparison to the all-stop results, the one-stop policy assigns more aircraft to each city on average. Also ExtraPassgr entries become smaller, while PropDirPassgr become larger. All these changes are due to the one-stop restriction, since it is more difficult to combine and coordinate passengers to take advantage of economies of scale.

5　Concluding Remarks

In this paper, we have proposed a new set of formulations for the problem of designing a capacitated airline networks. We have proposed heuristics and tested them on two data sets. The quality of the procedures have been shown to be excellent for problems in which the entries of the origin-destination demand matrix are large enough (say $f_{ij} \geq 20$).

With respect to the networks obtained by our procedures, our conclusions and observations are based on the analysis of heuristic solutions (as opposed to optimal ones), and

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thus must be interpreted as such. With this caveat, our main findings can be summarized as follows:

Given a fixed origin-destination demand matrix, an efficient design suggests the presence of strong connecting cities, which we can call hubs. However, the network structure is far from looking like a pure hub-and-spoke system (based on single or multiple allocation).

Given a set of cities and their relative positions, hub candidates depend more on their geographical position than on their own demand level. So it is quite likely that some cities will remain good hub candidates in a wide range of demand levels.

With a relatively high level of demand flows, the difference between the three policies is insignificant. One-stop policy could be as good as the two-stop policy. In practice, the one-stop policy is more service oriented and would be preferred, enabling the airline to gain higher market shares.

Finally the two-fleet option doesn't add a great advantage to an already efficient design (an average 1% cost reduction has been observed across problems). Also the cost reductions with a second fleet decrease as the demand level increases (from 14% in Problem 1 to 0.12% in Problem 6). Considering the additional operating costs, the adoption of a multiple fleet option really becomes questionable.

Acknowledgements:

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Table 1: Sample Cities and Their Populations

Index	City	Population	Index	City	Population
1	Albuquerque	480577	21	Lousville	952622
2	Atlanta	2833511	22	Memphis	981747
3	Austin	781572	23	Miami	3192582
4	Buffalo	1189288	24	Milwaukee	1607183
5	Baltimore	2382172	25	New Orleans	1238816
6	Boston	4171643	26	New York	18087251
7	Chicago	8065633	27	Oklahoma City	958839
8	Cincinnati	1744124	28	Philadelphia	2122101
9	Cleveland	2759824	29	Phoenix	5899345
10	Columbia	453331	30	Portland	1477895
11	Columbus	1377419	31	Richmond	865640
12	Dallas	3885415	32	Salt Lake City	1072227
13	Denver	1848319	33	San Antonio	1302099
14	Des Moines	392928	34	San Diego	2498016
15	Detroit	4665236	35	San Francisco	6253311
16	Houston	3711043	36	Seattle	2559164
17	Indianapolis	1249822	37	St Louis	2444099
18	Kansas City	1566280	38	St Paul	2464124
19	L.A.	14531529	39	Washington D.C.	3923574
20	Las Vegas	741459			

Table 2: Minimum and Maximum Demand Levels of the Test Problems

Problem Index	Minimum	Maximum
1	2	76
2	5	192
3	8	307
4	12	384
5	15	576
6	20	768

Table 3: Computational Test of the Heuristics

Pb size	Policy	Lower bound(lb)	1-fleet sol1	2-fleet sol2	Gap sol1	Gap sol2	CPU1 seconds	CPU2 seconds
CAB	one	111401.8	138136	131084.8	0.239	0.177	55.23	97.17
25	two	111401.8	124419	122429.6	0.117	0.099	492.78	483.86
	all	112348.3	118583	118386.6	0.055	0.054	341.49	494.89
1	one	52076.9	105703	82015.0	1.030	0.575	245.6	1358.8
39	two	52076.9	64943	63271.2	0.247	0.215	1441.9	4324.6
	all	52313.6	59038	58827.0	0.128	0.124	747.8	808.51
2	one	133464.6	151451	148110.7	0.135	0.110	770.3	1063.9
39	two	133464.6	147782	145012.2	0.107	0.086	1835.2	3054.9
	all	133592.1	143044	142671.2	0.071	0.068	1045.4	1101.8
3	one	215028.7	230650	225536.6	0.073	0.049	1464.1	1551.8
39	two	215028.7	226115	225353.0	0.051	0.048	5492.3	6743.2
	all	215385.5	226022	225073.0	0.049	0.045	1132.8	1294.9
4	one	269162.2	283242	279593.8	0.052	0.039	1650.4	2035.3
39	two	269162.2	279772	279162.7	0.039	0.037	2169.6	3521.4
	all	269272.6	279366	278618.3	0.037	0.034	1154.2	1288.5
5	one	404950.1	415614	413407.0	0.026	0.021	3934.4	4234.1
39	two	404950.1	414634	413325.7	0.024	0.021	5853.9	6489.6
	all	405156.5	412403	411919.4	0.018	0.017	1346.7	1465.6
6	one	540561.0	551669	550352.3	0.020	0.018	7299.2	7746.5
39	two	540561.0	551669	550352.3	0.020	0.018	8567.1	9502.4
	all	540769.8	548749	547991.0	0.014	0.013	1261.9	1387.9

Table 4: CAB, Two-Stop Policy, One-Fleet Option

City	Plane	ExtraPlane	OriPassgr	ExtraPassgr	PropDirPassgr
ATLANTA	7	3	652	304	31.29
BALTIMORE	**11**	**8**	**380**	**800**	**31.84**
BOSTON	8	0	1404	18	65.17
CHICAGO	14	1	2337	91	61.92
CINCINNATI	**13**	**11**	**352**	**994**	**68.75**
CLEVELAND	8	4	686	377	55.25
DALLAS-FW	7	3	705	277	42.27
DENVER	5	1	556	172	30.58
DETROIT	12	6	989	586	62.79
HOUSTON	7	3	549	356	37.70
KANSAS CITY	7	4	452	404	55.97
LOS ANGELES	10	0	1699	50	57.68
MEMPHIS	**12**	**10**	**258**	**951**	**63.95**
MIAMI	8	0	1284	78	49.30
MINNEAPOLIS	5	1	573	164	43.46
NEW ORLEANS	8	5	418	511	35.65
NEW YORK	22	0	3953	3	82.29
PHILADELPHIA	6	1	823	128	33.05
PHOENIX	7	5	335	462	67.76
PITTSBURGH	9	5	655	482	46.41
ST. LOUIS	5	1	666	117	31.68
SAN FRANCISCO	7	0	1173	43	48.08
SEATTLE	6	3	436	322	35.32
TAMPA	5	2	425	237	25.65
WASHINGTON	9	1	1326	147	59.95

Table 5: CAB, Strong Hub Candidates

Policy	1-fleet	2-fleet
one	Cincinnati, Memphis	Cincinnati, Memphis
two	Baltimore, Cincinnati, Memphis	Baltimore, Cincinnati, Memphis
all	Cincinnati, Denver, St. Louis	Cincinnati, Memphis, Phoenix

Table 6: CAB, p-Hub Solutions in Skorin-Kapov et al.

Allocation	Alpha	Three Hubs Solution
single	0.4	Chicago, Los Angeles, Philadelphia
	0.6	Baltimore, Chicago, Los Angeles
	0.8	Baltimore, Chicago, Los Angeles
multiple	0.4	Chicago, Los Angeles, New York
	0.6	Chicago, Los Angeles, New York
	0.8	Chicago, Los Angeles, New York

Table 7: 39-1-City, Two-Stop Policy, One-Fleet Option

City	Plane	ExtraPlane	OriPassgr	ExtraPassgr	PropDirPassgr
1	5	3	181	359	7.73
2	6	3	453	314	18.98
3	7	5	233	511	17.6
4	3	1	292	124	7.19
5	3	0	417	62	12.23
6	4	0	556	82	9.35
7	5	0	766	67	11.62
8	3	1	354	93	7.06
9	10	7	449	675	30.29
10	8	7	175	632	18.29
11	**13**	**11**	**313**	**1014**	**35.78**
12	4	1	532	94	6.95
13	3	0	368	86	4.35
14	1	0	160	0	1.88
15	7	3	586	337	27.82
16	5	2	522	189	10.34
17	7	5	299	481	18.06
18	**14**	**12**	**334**	**1083**	**41.32**
19	7	1	1013	124	20.63
20	8	6	225	608	24.00
21	8	6	261	590	21.46
22	9	7	264	678	23.86
23	3	0	486	27	4.12
24	4	2	339	191	10.03
25	3	1	298	121	7.05
26	9	2	1122	249	24.42
27	8	6	262	589	16.79
28	3	0	391	74	11.25
29	5	1	653	123	15.93
30	2	0	325	17	4.31
31	4	2	243	238	12.76
32	5	3	274	313	12.41
33	2	0	305	25	2.95
34	5	2	426	237	18.54
35	5	1	675	113	14.37
36	3	0	431	54	4.87
37	**14**	**11**	**424**	**1048**	**32.31**
38	3	0	426	57	5.4
39	4	1	535	93	12.71